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NEW CONCEPTS OF MICRO TOPOLOGICAL SPACES VIA MICRO IDEALS

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Abstract. The purpose of this paper is to introduced a new concept of spaces called micro ideal topological spaces and investigate the relation between micro topological space and micro ideal topological spaces. We define some closed sets in these spaces to establish their relationships. Basic properties and characterizations related to these sets are given. We introduced and studied the new concepts is called micro regular-open set, micro π-open set in micro topological spaces and also the new concepts called $m\mathcal{I}$-open, α-$m\mathcal{I}$-open, pre-$m\mathcal{I}$-open, semi-$m\mathcal{I}$-open, b-$m\mathcal{I}$-open, β-$m\mathcal{I}$-open, regular $m\mathcal{I}$-closed, which are simple forms of micro open sets in an micro ideal topological spaces. Also we characterize the relations between them and the related properties.

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1. INTRODUCTION

An ideal $\mathcal{I}$ on a topological space (X, τ) is a nonempty collection of subsets of X which satisfies (i) A $\in \mathcal{I}$ and B $\subset$ A $\Rightarrow$ B $\in \mathcal{I}$ and (ii) A $\in \mathcal{I}$ and B $\in \mathcal{I} \Rightarrow$ A $\cup$ B $\in \mathcal{I}$. Given a topological space (X, τ) with an ideal $\mathcal{I}$ on X and if $\wp$(X) is the set of all subsets of X, a set operator $(.)^* : \wp(X) \rightarrow \wp(X)$, called a local function [8] of A with respect to τ and $\mathcal{I}$ is defined as follows: for A $\subseteq$ X, $A^*(\mathcal{I}, \tau) = \{x \in X \mid U \cap A \notin \mathcal{I}$ for every $U \in \tau(x)\}$ where $\tau(x) = \{U \in \tau \mid x \in U\}$. We will make use of the basic facts about the local functions [[7], Theorem 2.3] without mentioning it explicitly. A Kuratowski closure operator $cl^*(.)$ for a topology $\tau^*(\mathcal{I}, \tau)$, called the $\star$-topology, finer than τ is defined by $cl^*(A) = A \cup A^*(\mathcal{I}, \tau)$ [10]. When there is no chance for confusion, we will simply write A^* for $A^*(\mathcal{I}, \tau)$ and τ^* for $\tau^*(\mathcal{I}, \tau)$.

If $\mathcal{I}$ is an ideal on X, then (X, τ, $\mathcal{I}$) is called an ideal space. $\mathcal{N}$ is the ideal of all nowhere dense subsets in (X, τ). A subset A of an ideal space (X, τ, $\mathcal{I}$) is $\star$-closed [7] (resp. $\star$-dense in itself [3]) if $A^* \subseteq A$ (resp. $A \subseteq A^*$).

By a space, we always mean a topological space (X, τ) with no separation properties assumed. If A $\subseteq$ X, cl(A) and int(A) will, respectively, denote the closure and interior of A in (X, τ) and $int^*(A)$ will denote the interior of A in (X, τ^*).

The notion of a micro topology was introduced and studied by Chandrasekar [1] which was defined micro closed, micro open, micro interior and micro closure. The purpose of this paper is to introduce a new concept of spaces called micro ideal topological spaces and investigate the relation between micro topological space and

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micro ideal topological spaces. We define some closed sets in these spaces to establish their relationships. Basic properties and characterizations related to these sets are studied. We introduce and study the new concept is called micro regular open set in micro topological spaces and also the new concepts called micro regular-open set, micro π-open set in micro topological spaces and also the new concepts called $m\mathcal{I}$-open, α-$m\mathcal{I}$-open, pre-$m\mathcal{I}$-open, semi-$m\mathcal{I}$-open, b-$m\mathcal{I}$-open, β-$m\mathcal{I}$-open, regular $m\mathcal{I}$-closed, which are simple forms of micro open sets in an micro ideal topological spaces. Also we characterize the relations between them and the related properties.

2. PRELIMINARIES

Definition 2.1. [9] *Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.*

(1) *The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$.*
$L_R(X) = \bigcup_{x \epsilon U}\{R(X)\colon R(X) \subseteq X\}$ *where $R(x)$ denotes the equivalence class determined by X.*

(2) *The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$.*
$U_R(X) = \bigcup_{x \epsilon U}\{R(X)\colon R(X) \cap X \neq \phi\}$

(3) *The boundary region of X with respect to R is the set of all objects, which can be neither in nor as not-X with respect to R and it is denoted by $B_R(X)$ and $B_R(X) = U_R(X) - L_R(X)$*

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Property 2.2. [9] *If (U, R) is an approximation space and X, $Y \subseteq U$, then*

(1) $L_R(X) \subseteq X \subseteq U_R(X)$.

(2) $L_R(\phi) = U_R(\phi) = \phi$, $L_R(U) = U_R(U) = U$.

(3) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$.

(4) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$.

(5) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$.

(6) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$.

(7) $L_R(X) \subseteq L_R(Y)$ *and* $U_R(X) \subseteq U_R(Y)$, *whenever* $X \subseteq Y$.

(8) $U_R(X^c) = [L_R(X)]^c$ *and* $L_R(X^c) = [U_R(X)]^c$.

(9) $U_R(U_R(X)) = L_R(U_R(X)) = U_R(X)$.

(10) $L_R(L_R(X)) = U_R(L_R(X)) = L_R(X)$.

Definition 2.3. [9] *Let U be an universe, R be an equivalence relation on U and* $\tau_R(X) = \{U, \phi , L_R(X), U_R(X), B_R(X)\} where X \subseteq U$. *Then by Property 2.2,* $\tau_R(X)$ *satisfies the following axioms*

(1) U , $\phi \epsilon \tau_R(X)$.

(2) *The union of the elements of any sub-collection of* $\tau_R(X)$ *is in* $\tau_R(X)$.

(3) *The intersection of the elements of any finite sub collection of* $\tau_R(X)$ *is in* $\tau_R(X)$.

Then $\tau_R(X)$ *is called the nano topology on U with respect to X.*

The space $(U, \tau_R(X))$ *is the nano topological space. The elements of are called nano open sets.*

Definition 2.4. [9]

If $(U, \tau_R(X))$ *is the nano topological space with respect to X where* $X \subseteq U$ *and if* $A \subseteq U$, *then*

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(1) *The nano interior of the set A is defined as the union of all nano open subsets contained in A and it is denoted by nint(A). That is, nint(A) is the largest nano open subset of A.*
(2) *The nano closure of the set A is defined as the intersection of all nano closed sets containing A and it is denoted by ncl(A). That is, ncl(A) is the smallest nano closed set containing A.*

Definition 2.5. [1] *Let (U, $\tau_R(X)$) be a nano topological space. Then, $\mu_R(X) = \{N \cup (\acute{N} \cap \mu) : N, \acute{N} \in \tau_R(X)$ and $\mu \notin \tau_R(X)\}$ is called the micro topology on U with respect to X. The triplet (U, $\tau_R(X)$, $\mu_R(X)$) is called micro topological space and the elements of $\mu_R(X)$ are called micro open sets and the complement of a micro open set is called a micro closed set.*

Definition 2.6. [1] *The micro topology $\mu_R(X)$ satises the following axioms*

(1) *U, $\phi \in \mu_R(X)$.*
(2) *The union of the elements of any sub-collection of $\mu_R(X)$ is in $\mu_R(X)$.*
(3) *The intersection of the elements of any finite sub collection of $\mu_R(X)$ is in $\mu_R(X)$.*

Then $\mu_R(X)$ is called the micro topology on U with respect to X. The triplet (U, $\tau_R(X)$, $\mu_R(X)$) is called micro topological spaces and The elements of $\mu_R(X)$ are called micro open sets and the complement of a micro open set is called a micro closed set.

Definition 2.7. [1] *The micro interior of a set A is denoted by micro-int(A) and is defined as the union of all micro open sets contained in A. i.e., Mic-int(A) $= \cup \{G$: G is micro open and $G \subseteq A \}$.*

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Definition 2.8. [1] *The micro closure of a set A is denoted by micro-cl(A) and is defined as the intersection of all micro closed sets containing A. i.e., Mic-cl(A)* $= \cap \{F : F$ *is micro closed and* $A \subseteq F\}$.

Definition 2.9. [1] *For any two micro sets A and B in a micro topological space* $(U, \tau_R(X), \mu_R(X))$,

(1) *A is a micro closed set if and only if Mic-cl(A) = A.*

(2) *A is a micro open set if and only if Mic-int(A) = (A).*

(3) $A \subseteq B$ *implies Mic-int(A)* $\subseteq$ *Mic-int(B) and Mic-cl(A)* $\subseteq$ *Mic-cl(B).*

(4) *Mic-cl(Mic-cl(A)) = Mic-cl(A) and Mic-int(Mic-int(A)) = Mic-int(A).*

(5) *Mic-cl* $(A \cup B) \supseteq$ *Mic-cl(A)* $\cup$ *Mic-cl(B).*

(6) *Mic-cl*$(A \cap B) \subseteq$ *Mic-cl(A)* $\cap$ *Mic-cl(B).*

(7) *Mic-int*$(A \cup B) \supseteq$ *Mic-int(A)* $\cup$ *Mic-int(B).*

(8) *Mic-int*$(A \cap B) \subseteq$ *Mic-int(A)* $\cap$ *Mic-int(B).*

(9) *Mic-cl*$(\mathcal{A}^C) = [Mic - int(A)]^C$.

(10) *Mic-int*$(A^C) = [Mic - cl(A)]^C$.

Definition 2.10. *Let* $(U, \tau_R(X), \mu_R(X))$ *be a micro topological space and* $A \subseteq U$. *Then,*

(1) *A is called micro* α*-open if* $A \subseteq$ *Mic-int(Mic-cl(Mic-int(A)))* [2].

(2) *A is called micro pre-open if* $A \subseteq$ *Mic-int(Mic-cl(A))* [1].

(3) *A is called micro semi-open if* $A \subseteq$ *Mic-cl(Mic-int(A))* [1].

(4) *A is called micro b-open if* $A \subseteq$ *Mic-int(Mic-cl(A))* $\cup$ *Mic-cl(Mic-int(A))*[5, 6].

(5) *A is called micro* β*-open if* $A \subseteq$ *Mic-cl(Mic-int(Mic-cl(A))).* [5]

The complement of above mentioned micro open sets are called their respective micro closed sets.

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The family of all micro α-open (resp. micro pre-open, micro semi-open, micro b-open, micro β-open) sets in (U, $\tau_R(X)$, $\mu_R(X)$) is denoted by MαO (U, $\tau_R(X)$, $\mu_R(X)$)(resp. MPO (U, $\tau_R(X)$, $\mu_R(X)$), MSO (U, $\tau_R(X)$, $\mu_R(X)$), MbO (U, $\tau_R(X)$, $\mu_R(X)$), MβO (U, $\tau_R(X)$, $\mu_R(X)$)).

Definition 2.11. *Let (K, $\tau_R(X)$, $\mu_R(X)$) be a micro topological space and $A \subseteq K$. Then A is called micro regular open if A = Mic-int(Mic-cl(A)). The complement of micro regular open sets is called micro regular closed sets.*
The family of all micro regular open set in (K, $\tau_R(X)$, $\mu_R(X)$) is denoted by MRO (k, $\tau_R(X)$, $\mu_R(X)$).

Example 2.12. *Let K = $\{1, 2, 3\}$ with K/ R= $\{\{2\}, \{1, 3\}\}$ and X= $\{2\}$. The nano topology $\tau_R(X) = \{\phi, \{2\}, K\}$. If $\mu = \{1, 3\}$ then the micro topology $\mu_R(X) = \{\phi, \{2\}, \{1, 3\}, K\}$. The micro closed sets are ϕ, $\{2\}$, $\{1, 3\}$, K. Also Mic-int(Mic-cl(A)) = A for A = ϕ, $\{2\}$, $\{1, 3\}$, K and hence these sets are micro regular open in K.*

Theorem 2.13. *Any micro regular open set is micro open.*

Proof. If A is micro regular open in (K, $\tau_R(X)$, $\mu_R(X)$), A = Mic-int(Mic-cl(A)). Then Mic-int(A) = Mic-int(Mic-int(Mic-cl(A))) = Mic-int(Mic-cl(A)) = A. That is, Ais micro open in K.

Example 2.14. *Let K = $\{1, 2, 3, 4, 5\}$ with K/ R= $\{\{3\}, \{5\}, \{1, 2\}\}$ and X= $\{1, 2\}$. The nano topology $\tau_R(X) = \{\phi, \{1, 2\}, K\}$. If $\mu = \{4\}$ then the micro topology $\mu_R(X) = \{\phi, \{4\}, \{1, 2\}, \{1, 2, 4\}\ K\}$. The micro closed sets are ϕ, $\{3, 5\}$, $\{3, 4, 5\}$, $\{1, 2, 3, 5\}$, K. The micro regular open sets are ϕ, $\{4\}$, $\{1, 2\}$, K. Thus, we note that $\{1, 2, 4\}$ is micro open but not micro regular open. Also, we note that the micro* *regular open sets do not form a micro topology, since* $\{4\} \cup \{1, 2\} = \{1, 2, 4\}$ *is not micro regular open, but* $\{4\}$ *and* $\{1, 2\}$ *are micro regular open.*

Definition 2.15. *Let A subset A of an micro topological space* $(U, \tau_R(X), \mu_R(X))$ *is said to be micro* π*-open set if A is the finite union of micro regular-open sets.*

Remark 2.16. *For a subset of A of an micro topological space* $(U, \tau_R(X), \mu_R(X))$*, we have following implications:*

micro regular open $\Rightarrow$ *micro* π*-open* $\Rightarrow$ *micro open*

Diagram-I

Example 2.17. *Let* $K = \{1, 2, 3\}$ *with* $K/R = \{\{1\}, \{2, 3\}\}$ *and* $X = \{1\}$*. The nano topology* $\tau_R(X) = \{\phi, \{1\}, K\}$*. If* $\mu = \{2\}$ *then the micro topology* $\mu_R(X) = \{\phi, \{1\}, \{2\}, \{1, 2\}, K\}$*. Then micro* π*-open sets are* $\phi, \{1\}, \{2\}, \{1, 2\}, K$ *; micro regular open sets are* $\phi, \{1\}, \{2\}, K$*. It is clear that* $\{1, 2\}$ *is micro* π*-open set but it is not micro regular open set.*

Example 2.18. *Let* $K = \{1, 2, 3\}$ *with* $K/R = \{\{1\}, \{2, 3\}\}$ *and* $X = \{1, 2\}$*. The nano topology* $\tau_R(X) = \{\phi, \{1\}, \{2, 3\}, K\}$*. If* $\mu = \{2\}$ *then the micro topology* $\mu_R(X) = \{\phi, \{1\}, \{2\}, \{1, 2\}, \{2, 3\}, K\}$*. Then micro* π*-open sets are* $\phi, \{1\}, \{2, 3\}, K$*. It is clear that* $\{1, 2\}$ *is micro open set but it is not micro* π*-open set.*

3. micro Ideal Topological spaces

In 1990, Jankovic and Hamlett [[4], [7]] have considered the local function in ideal topological space any they have obtained a new topology. In this section we shall introduce a similar type with the local function in micro topological spaces. Before starting the discussion we shall consider the following concepts.

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Let (K, $\mathcal{N}$, $\mathcal{M}$) be a micro topological space , where $\mathcal{N} = \tau_R(X)$ and $\mathcal{M} = \mu_R(X)$. A micro topological space (K, $\mathcal{N}$, $\mathcal{M}$) with an ideal $\mathcal{I}$ on K is called a micro ideal topological space and is denoted by (K, $\mathcal{N}$, $\mathcal{M}$, $\mathcal{I}$).

Let (K, $\mathcal{M}$) be a micro topological space and $S_m(k) = \{S_m \mid k \in S_m, S_m \in \mathcal{M}\}$ be the family of micro open sets which contain k.

Definition 3.1. *Let (K, $\mathcal{N}$, $\mathcal{M}$) be a micro topological space with an ideal $\mathcal{I}$ on K and if $\wp(K)$ is the set of all subsets of K, a set operator $(.)^*_m : \wp(K) \to \wp(K)$. For a subset $A \subset K$, $A^*_m(\mathcal{I}, \mathcal{M}) = \{k \in K : S_m \cap A \notin \mathcal{I}$, for every $S_m \in S_m(k)\}$ is called the micro local function (brielfy, m-local function) of A with repect to $\mathcal{I}$ and $\mathcal{M}$. We will simply write A^*_m for $A^*_m(\mathcal{I}, \mathcal{M})$.*

Example 3.2. *(K, $\mathcal{N}$, $\mathcal{M}$) be a micro topological space with an ideal $\mathcal{I}$ on K and for every $A \subseteq K$.*

(1) *If $\mathcal{I} = \{\phi\}$ then $A^*_m = m\text{-}cl(A)$,*
(2) *If $\mathcal{I} = P(K)$, then $A^*_m = \phi$.*

Theorem 3.3. *Let (K, $\mathcal{N}$, $\mathcal{M}$) be a micro topological space with ideal $\mathcal{I}, \mathcal{I}'$ on K and A, B be subsets of K. Then*

(1) $A \subseteq B \Rightarrow A^*_m \subseteq B^*_m$,
(2) $\mathcal{I} \subseteq \mathcal{I}' \Rightarrow A^*_m(\mathcal{I}') \subseteq A^*_m(\mathcal{I})$,
(3) $A^*_m = m\text{-}cl(A^*_m) \subseteq m\text{-}cl(A)$ (A^*_m *is a micro closed subset of* m-cl(A)),
(4) $(A^*_m)^*_m \subseteq A^*_m$,
(5) $A^*_m \cup B^*_m = (A \cup B)^*_m$,
(6) $A^*_m - B^*_m = (A - B)^*_m - B^*_m \subseteq (A - B)^*_m$,
(7) $U \in \mathcal{M} \Rightarrow U \cap A^*_m = U \cap (U \cap A)^*_m \subseteq (U \cap A)^*_m$ *and*

(8) *$F \in \mathcal{I} \Rightarrow (A \cup F)^*_m = A^*_m = (A - F)^*_m$, and so $A^*_m = \phi$, if $A \in \mathcal{I}$.*

Proof. (1) Let A $\subseteq$ B and k $\in$ A^*_m. Assume that k $\notin$ B^*_m. We have $S_m \cap B \in \mathcal{I}$ for some $S_m \in S_m(k)$. Since $S_m \cap A \subseteq S_m \cap B$ and $S_m \cap B \in \mathcal{I}$, we obtain $S_m \cap A \in \mathcal{I}$ from the definition of ideal. Thus, we have k $\notin$ A^*_m. This is a contradiction. Clearly, $A^*_m \subseteq B^*_m$.

(2) Let $\mathcal{I} \subseteq \mathcal{I}'$ and k $\in$ A^*_m $(\mathcal{I}')$. Then we have $S_m \cap A \notin \mathcal{I}'$ for every $S_m \in S_m(k)$. By hypothesis, we obtain $S_m \cap A \notin \mathcal{I}$. So k $\in$ A^*_m $(\mathcal{I})$.

(3) Let k $\in$ A^*_m. Then for every $S_m \in S_m(k)$, $S_m \cap A \in \mathcal{I}$. This implies that $S_m \cap A \neq \phi$. Hence k $\in$ m-cl(A).

(4) From (3), $(A^*_m)^*_m \subseteq$ m-cl$(A^*_m) = A^*_m$, since A^*_m is a micro closed set.

The proofs of the other conditions are also obvious. □

Example 3.4. *(1) Let $K = \{1, 2, 3, 4\}$ with $K/R = \{\{1\}, \{3\}, \{2, 4\}\}$ and $X = \{2, 4\}$. The nano topology $\mathcal{N} = \{\phi, \{2, 4\}, K\}$. If $\mu = \{1\}$ then the micro topology $\mathcal{M} = \{\phi, \{1\}, \{2, 4\}, \{1, 2, 4\}, K\}$ and the ideal $\mathcal{I} = \{\emptyset, \{1\}\}$. For, $A = \{1, 3\}$ and $B = \{1, 4\}$, we have $A^*_m = \{3\} \subset B^*_m = \{2, 3, 4\}$ but $A \nsubseteq B$.*

(2) Let K, $\mathcal{N}$, μ, $\mathcal{M}$ be defined as an Example 3.4 (1), and the ideal $\mathcal{I} = \{\emptyset, \{2\}\}$. It is easily seen that $\mathcal{I} \nsubseteq \mathcal{I}'$. For $A = \{1, 3, 4\}$. A^*_m $(\mathcal{I}) = \{2, 3, 4\} \subset A^*_m$ $(\mathcal{I}') = K$.

(3) Let $K = \{1, 2, 3, 4\}$ with $K/R = \{\{3\}, \{4\}, \{1, 2\}\}$ and $X = \{1, 2\}$. The nano topology $\mathcal{N} = \{\phi, \{1, 2\}, K\}$. If $\mu = \{3\}$ then the micro topology $\mathcal{M} = \{\phi, \{3\}, \{1, 2\}, \{1, 2, 3\}, K\}$ and the ideal $\mathcal{I} = \{\emptyset, \{1\}\}$. For, $A = \{1, 4\}$, we have m-cl(A) $= \{1, 2, 4\}$, $A^*_m = \{4\}$ and m-cl$(A^*_m) = \{4\}$. Therefore, m-cl(A) $\nsubseteq A^*_m =$ m-cl(A^*_m).

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Theorem 3.5. *Let (K, $\mathcal{N}$, $\mathcal{M}$) be an micro topological space with an ideal $\mathcal{I}$ and A $\subseteq A^*_m$, then $A^*_m = m\text{-}cl(A^*_m) = m\text{-}cl(A)$.*

Proof. For every subset A of K, we have A^*_m = m-cl(A^*_m) = m-cl(A), by Theorem 3.3 (3). A $\subseteq A^*_m$ implies that m-cl(A) $\subseteq$ m-cl(A^*_m) and so A^*_m = m-cl(A^*_m) = m-cl(A). □

Definition 3.6. *Let (K, $\mathcal{N}$, $\mathcal{M}$) be an micro topological space with an ideal $\mathcal{I}$ on K. The set operator m-cl* is called a micro $\star$-closure and is defined as m-cl*(A) = A $\cup$ A^*_m for A $\subseteq$ K.*

Theorem 3.7. *The set operator m-cl* satisfies the following conditions:*

(1) *A $\subseteq$ m-cl*(A),*
(2) *m-cl*(ϕ) = ϕ and m-cl*(K) = K,*
(3) *If A $\subseteq$ B, then m-cl*(A) $\subseteq$ m-cl*(B),*
(4) *m-cl*(A) $\cup$ m-cl*(B) = m-cl*(A $\cup$ B),*
(5) *m-cl*(m-cl*(A)) = m-cl*(A).*

Proof. The proofs are clear from Theorem 3.3 and Definition 3.6. □

Now, $\mathcal{M}^*$ ($\mathcal{I}$, $\mathcal{M}$) = { U $\subset$ K; m-cl*(K − U) = K − U}. $\mathcal{M}^*$ ($\mathcal{I}$, $\mathcal{M}$) is called micro $\star$-topology which is finer than $\mathcal{M}$ (we simply write $\mathcal{M}^*$ for $\mathcal{M}^*$ ($\mathcal{I}$, $\mathcal{M}$). The elements of $\mathcal{M}^*$ ($\mathcal{I}$, $\mathcal{M}$) are called micro $\star$-open (briefly, $m\star$-open) set and the complement of an $m\star$-open set is called is called micro $\star$-closed (briefly, $m\star$-closed) set. Here m-cl*(A) and m-int*(A) will denote the closure and interior of A in (K, $\mathcal{M}^*$).

Remark 3.8. *(i) See Example 3.2 (1), if $\mathcal{I} = \{\phi\}$ then A^*_m = m-cl(A). In this case, m-cl*(A) = m-cl(A).*

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(ii) If $(K, \mathcal{N}, \mathcal{M}, \mathcal{I})$ is a micro ideal topological space with $\mathcal{I} = \{\phi\}$, then $\mathcal{M}^* = \mathcal{M}$.

Definition 3.9. *A basis $\beta(\mathcal{I}, \mathcal{M})$ for $\mathcal{M}^*$ can be described as follows: $\beta(\mathcal{I}, \mathcal{M}) = \{V - F\colon V \in \mathcal{M}, F \in \mathcal{I}\}$.*

Lemma 3.10. *Let $(K, \mathcal{N}, \mathcal{M})$ be a micro topological space and $\mathcal{I}$ be an ideal on K. Then $\beta(\mathcal{I}, \mathcal{M})$ is a basis for $\mathcal{M}^*$.*

Proof. Straight forward. □

We have to show that for a given space (K, $\mathcal{N}$, $\mathcal{M}$) and an ideal $\mathcal{I}$ on K, $\beta(\mathcal{I}, \mathcal{M})$ is a basis for $\mathcal{M}^*$. If $\beta(\mathcal{I}, \mathcal{M})$ is itself a micro topology, then we have If $\beta(\mathcal{I}, \mathcal{M}) = \mathcal{M}^*$ and all the open sets of $\mathcal{M}^*$ are of simple form V − F where V $\in \mathcal{M}$ and F $\in \mathcal{I}$.

Lemma 3.11. *If $(K, \mathcal{N}, \mathcal{M})$ is a micro topological space and $\mathcal{I}$ be an ideal on K and if $F \in \mathcal{I}$, then F is m$\star$-closed and if $A \subset K$ is m$\star$-closed then $A^*_m \subseteq A$.*

Theorem 3.12. *$(K, \mathcal{N}, \mathcal{M})$ be an micro topological space with an ideal $\mathcal{I}$ on K and for every $A \subseteq K$. If $A \subseteq A^*_m$, then*

(1) *m-cl(A) = m-cl*(A),*
(2) *m-int(K − A) = m-int*(K − A),*
(3) *m-cl(K − A) = m-cl*(K − A),*
(4) *m-int(A) = m-int*(A).*

Proof. (1) Follows immediately from Theorem 3.5.

(2) If A $\subseteq A^*_m$ then m-cl(A) = m-cl*(A) by (1) and so K − m-cl(A) = K − m-cl*(A). Therefore, m-int(K − A) = m-int*(K − A).

(3) Follows by replacing A by K − A in(1).

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(4) If A ⊆ A^*_m then m-cl(K − A) = m-cl*(K − A) by (3) and so K − m-cl(K − A) = K − m-cl*(K − A). Therefore, m-int(A) = m-int*(A). □

Theorem 3.13. *(K, $\mathcal{N}$, $\mathcal{M}$) be an micro topological space with an ideal $\mathcal{I}$ on K and for every A ⊆ K. If A ⊆ A^*_m, then A^*_m = m-cl(A^*_m) = m-cl(A) = m-cl*(A).*

Proof. Follows from Thoerem 3.5 and Theorem 3.12(1). □

Definition 3.14. *A subset A of a micro ideal topological space (K, $\mathcal{N}$, $\mathcal{M}$, $\mathcal{I}$) is m⋆ dense in itself (resp. m⋆-perfect, m⋆-closed) if A ⊆ A^*_m (resp. A = A^*_m, A^*_m ⊆ A).*

Remark 3.15. *we have the following diagram*

m⋆ dense in itself ⇐ m⋆-perfect ⇒ m⋆-closed

Diagram-II

The following examples show that the converse implications of the diagram are not satisfied.

Example 3.16. *Let K, $\mathcal{N}$, μ, $\mathcal{M}$ be defined as an Example 3.4 (1). For (i) A = {1, 3}, we have A^*_m = {3}. Here A is m⋆-closed but not m⋆-perfect.*
(ii) A = {2, 3}, we have A^*_m = {2, 3, 4}. Here A is m⋆ dense in itself but not m⋆-perfect.

Theorem 3.17. *(K, $\mathcal{N}$, $\mathcal{M}$) be an micro topological space with an ideal $\mathcal{I}$ on K and for every A ⊆ K. If A is m⋆ dense in itself, then A^*_m = m-cl(A^*_m) = m-cl(A) = m-cl*(A).*

Proof. Let A be m⋆ dense in itself. Then we have A ⊆ A^*_m and using Theorem 3.13, we get A^*_m = m-cl(A^*_m) = m-cl(A) = m-cl*(A). □

Lemma 3.18. *(K, $\mathcal{N}$, $\mathcal{M}$) be an micro topological space with an ideal $\mathcal{I}$ on K and for every A ⊆ K then $A^*_m(\mathcal{I}, \mathcal{M}) = A^*_m(\mathcal{I}, \mathcal{M}^*)$ and hence $\mathcal{M}^* = \mathcal{M}^{**}$.*

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The study of ideal got new dimension when codence ideal [7] has been incorporated in ideal topological space. Now we introduce similar concept in micro ideal topological spaces.

Definition 3.19. *An ideal $\mathcal{I}$ in a space $(K, \mathcal{N}, \mathcal{M}, \mathcal{I})$ is called $\mathcal{M}$-codense ideal if $\mathcal{M} \cap \mathcal{I} = \{\phi\}$.*

Theorem 3.20. *Let $(K, \mathcal{N}, \mathcal{M}, \mathcal{I})$ be an micro ideal topological space and $\mathcal{I}$ is $\mathcal{M}$-codense with $\mathcal{M}$. Then $K = K^*_m$.*

Proof. It is obvious that $K^*_m \subseteq K$. For converse, suppose $k \in K$ but $k \notin K^*_m$. Then there exists $S_k \in \mathcal{M}(k)$ such that $S_k \cap K \in \mathcal{I}$. That is $S_k \in \mathcal{I}$, a contradiction to the fact that $\mathcal{M} \cap \mathcal{I} = \{\phi\}$. Hence $K = K^*_m$. □

Lemma 3.21. *If $(K, \mathcal{N}, \mathcal{M}, \mathcal{I})$ is any micro ideal topological space, then the following are equivalent*

(1) $K = K^*_m$,
(2) $\mathcal{M} \cap \mathcal{I} = \{\phi\}$,
(3) *If $F \in \mathcal{I}$ then m-int(F) $= \phi$,*
(4) *for every $G \in \mathcal{M}$, $G \subseteq G^*_m$.*

Proof. $A^*_m(\mathcal{I}, \mathcal{M}) = A^*_m(\mathcal{I}, \mathcal{M}^*)$ [by Lemma 3.18], we many replace $\mathcal{M}$ by $\mathcal{M}^*$ in (2), m-int(F) $= \phi$ by m-int$^*(F) = \phi$ in (3) and $G \in \mathcal{M}$ by $G \in \mathcal{M}^*$ in (4). □.

Lemma 3.22. *If $(K, \mathcal{N}, \mathcal{M})$ be a micro topological space with an ideal $\mathcal{I}$ on K, then the following are equivalent.*

(1) *MRO$(K, \mathcal{N}, \mathcal{M}) \cap \mathcal{I} = \{\phi\}$,*
(2) $\mathcal{M} \cap \mathcal{I} = \{\phi\}$.

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Proof. Since MRO(K, $\mathcal{N}$, $\mathcal{M}$) $\subset \mathcal{M}$, it is enough to prove that (1) $\Rightarrow$ (2). Suppose A $\in \mathcal{M} \cap \mathcal{I}$. By Lemma 3.21 (3), A $\in \mathcal{I}$ implies that m-int(A) $= \phi$ and so, since A $\in \mathcal{M}$, A $= \phi$. Therefore, $\mathcal{M} \cap \mathcal{I} = \{\phi\}$. □

Lemma 3.23. *If (K, $\mathcal{N}$, $\mathcal{M}$) be a micro topological space with an ideal $\mathcal{I}$ on K, then the following are equivalent.*

(1) $\mathcal{M} \cap \mathcal{I} = \{\phi\}$,
(2) *MSO(K, $\mathcal{N}$, $\mathcal{M}$)* $\cap \mathcal{I} =\{\phi\}$,

Proof. Since $\mathcal{M} \subset$ MSO (K, $\mathcal{N}$, $\mathcal{M}$), it is enough to prove that (1) $\Rightarrow$ (2). Suppose A $\in$ MSO(K, $\mathcal{N}$, $\mathcal{M}$) $\cap \mathcal{I}$. By Lemma 3.21 (3), A $\in \mathcal{I}$ implies that m-int(A) $= \phi$ and so, since A $\in$ MSO(K, $\mathcal{N}$, $\mathcal{M}$), A $= \phi$. Therefore, MSO(K, $\mathcal{N}$, $\mathcal{M}$) $\cap \mathcal{I} = \{\phi\}$. □

Theorem 3.24. *If (K, $\mathcal{N}$, $\mathcal{M}$, $\mathcal{I}$) is any micro ideal topological space, then the following are equivalent*

(1) $K = K^*_m$,
(2) *for every* $A \in \mathcal{M}$, $A \subseteq A^*_m$,
(3) *for every* $A \in$ *MSO(K, $\mathcal{N}$, $\mathcal{M}$)*, $A \subseteq A^*_m$,
(4) *For every micro regular closed set F,* $F = F^*_m$.

Proof. (1) and (2) are equivalent by Lemma 3.21.
(2) $\Rightarrow$ (3). Suppose A $\in$ MSO(K, $\mathcal{N}$, $\mathcal{M}$). Then there exists an micro open set P such that P $\subseteq$ A $\subseteq$ m-cl(P). Since P is micro open, P $\subseteq$ P^*_m and so by Theorem 3.5, A $\subseteq$ m-cl(P) $\subseteq$ m-cl(P^*_m) $=$ P^*_m $\subseteq$ A^*_m. Hence A $\subseteq$ A^*_m.
(3) $\Rightarrow$ (4). If F is micro regular closed then F is micro semi oepn and micro closed. Fis micro semi open $\Rightarrow$ F $\subseteq$ F^*_m. F is micro closed implies that F is m$\star$-closed and

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so $F^*_m \subseteq F$, by Lemma 3.11. Hence $F = F^*_m$.

(4) ⇒ (1). It is clear. □

4. **Forms of m-open sets in (K, $\mathcal{N}$, $\mathcal{M}$, $\mathcal{I}$)**

Definition 4.1. *Let A subset A of an micro ideal topological space (K, $\mathcal{N}$, $\mathcal{M}$, $\mathcal{I}$) is said to be,*

(1) *micro $\mathcal{I}$-open (briefly, m$\mathcal{I}$-open) if $A \subseteq$ m-int(A^*_m).*
(2) *micro α-$\mathcal{I}$-open (briefly, α-m$\mathcal{I}$-open) if $A \subseteq$ m-int(m-cl*(m-int(A))).*
(3) *micro pre-$\mathcal{I}$-open (briefly, pre-m$\mathcal{I}$-open) if $A \subseteq$ m-int(m-cl*(A)).*
(4) *micro semi-$\mathcal{I}$-open (briefly, semi-m$\mathcal{I}$-open) if $A \subseteq$ m-cl*(m-int(A)).*
(5) *micro b-$\mathcal{I}$-open (briefly, b-m$\mathcal{I}$-open) if $A \subseteq$ m-int(m-cl*(A)) $\cup$ m-cl*(m-int(A)).*
(6) *micro β-$\mathcal{I}$-open (briefly, β-m$\mathcal{I}$-open) if $A \subseteq$ m-cl*(m-int(m-cl*(A))).*
(7) *micro α^*-$\mathcal{I}$-open (briefly, α^*-m$\mathcal{I}$-open) if m-int(A) = m-int(m-cl*(m-int(A))).*

The complement of above mentioned micro ideal open sets are called their respective micro ideal closed sets.

Theorem 4.2. *For a subset of an micro ideal topological space (K, $\mathcal{N}$, $\mathcal{M}$, $\mathcal{I}$) the following hold:*

(1) *Every α-m$\mathcal{I}$-open set is micro α-open.*
(2) *Every semi-m$\mathcal{I}$-open set is micro semi-open.*
(3) *Every β-m$\mathcal{I}$-open set is micro β-open.*
(4) *Every pre-m$\mathcal{I}$-open set is micro pre-open.*
(5) *Every b-m$\mathcal{I}$-open set is micro b-open.*

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Proof. (1) Let A be an α-$m\mathcal{I}$-open set. Then we have A $\subseteq$ m-int(m-cl*(m-int(A))) = m-int((m-int(A))$^*_m$ $\cup$ m-int(A)) $\subseteq$ m-int(m-int(A))$^*_m$ $\cup$ m-int(m-int(A)) $\subseteq$ m-int(m-int(A))$^*_m$ $\cup$ (m-int(A)) $\subseteq$ m-int(m-cl*(m-int(A) $\subseteq$ m-int(m-cl(m-int(A))) = Mic-int(Mic-cl(Mic-int(A))). Hence A is micro α-open.

(2) Let A be an semi-$m\mathcal{I}$-open set. Then we have A $\subseteq$ m-cl*(m-int(A)) $\subseteq$ (m-int(A))$^*_m$ $\cup$ (m-int(A)) $\subseteq$ m-cl*(m-int(A)) $\subseteq$ m-cl(m-int(A)) = Mic-cl(Mic-int(A)). Hence A is micro semi-open.

(3) Let A be an β-$m\mathcal{I}$-open set. Then we have A $\subseteq$ m-cl*(m-int(m-cl*(A))) $\subseteq$ m-cl*(m-int(A $\cup$ A$^*_m$)) $\subseteq$ m-cl*(m-int(A $\cup$ m-cl(A))) $\subseteq$ m-cl*(m-int(m-cl(A) $\cup$ m-cl(A)))$\subseteq$ m-cl*(m-int(m-cl(A))) $\subseteq$ ((m-int(m-cl(A)))$^*_m$ $\cup$ (m-int(m-cl(A)) $\subseteq$ m-cl(m-int(m-cl(A))) = Mic-cl(Mic-int(Mic-cl(A))). Hence A is micro β-open.

(4) Let A be an pre-$m\mathcal{I}$-open set. Then we have A $\subseteq$ m-int(m-cl*(A)) $\subseteq$ m-int(m-cl(A)) = Mic-int(Mic-cl(A)). Hence A is micro pre-open.

(5) Let A be an b-$m\mathcal{I}$-open set. Then we have A $\subseteq$ m-int(m-cl*(A)) $\cup$ m-cl*(m-int(A)) $\subseteq$ m-int(m-cl(A)) $\cup$ ((m-int(A))$^*_m$ $\cup$ m-int(A)) $\subseteq$ m-int(m-cl(A)) $\cup$ (m-cl*(m-int(A))) $\subseteq$ m-int(m-cl(A)) $\cup$ (m-cl(m-int(A)). Hence A is micro b-open. □

Theorem 4.3. *For a subset of an micro ideal topological space (K, $\mathcal{N}$, $\mathcal{M}$, $\mathcal{I}$) the following hold:*

(1) *Every α-$m\mathcal{I}$-open set is pre-$m\mathcal{I}$-open.*
(2) *Every α-$m\mathcal{I}$-open set is semi-$m\mathcal{I}$-open.*
(3) *Every pre-$m\mathcal{I}$-open set is b-$m\mathcal{I}$-open.*
(4) *Every pre-$m\mathcal{I}$-open set is β-$m\mathcal{I}$-open.*
(5) *Every b-$m\mathcal{I}$-open set is β-$m\mathcal{I}$-open.*
(6) *Every semi-$m\mathcal{I}$-open set is b-$m\mathcal{I}$-open.*
(7) *Every $m\mathcal{I}$-open set is pre-$m\mathcal{I}$-open.*

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Proof. The proof is obvious. □

Example 4.4. *Let K, $\tau_R(X)$, μ, $\mu_R(X)$ be defined as an Example 3.4 (1). Then (i) Here, A = {1, 2} is pre-m$\mathcal{I}$-open set but not α-m$\mathcal{I}$-open set. (ii) Here, B = {2, 3, 4} is semi-m$\mathcal{I}$-open set but not α-m$\mathcal{I}$-open set. (iii) Here, C = {2, 3, 4} is b-m$\mathcal{I}$-open set but not pre-m$\mathcal{I}$-open set. (iv) Here, D = {2, 3} is β-m$\mathcal{I}$-open set but not pre-m$\mathcal{I}$-open set. (v) Here, E = {3, 4} is β-m$\mathcal{I}$-open set but not b-m$\mathcal{I}$-open set. (vi) Here, F = {2} is b-m$\mathcal{I}$-open set but not semi-m$\mathcal{I}$-open set. (vii) Here, G = {1, 4} is pre-m$\mathcal{I}$-open set but not m$\mathcal{I}$-open set.*

Proposition 4.5. *Every micro open set of an micro ideal topological space is α-m$\mathcal{I}$-open.*

Proof. Let A be any micro open set. Then we have A = m-int(A). But A ⊆ m-cl*(A) ⊆ m-int(m-cl*(A)) ⊆ m-int(m-cl*(m-int(A))). Hence A is α-m$\mathcal{I}$-open. □

Example 4.6. *Let K = {1, 2, 3, 4, 5} with K/ R = {{1}, {4}, {2, 3}} and X = {2, 3}. The nano topology $\mathcal{N}$ = {ϕ, {2, 3}, K}. If μ = {1} then the micro topology $\mathcal{M}$ = {ϕ, {1}, {2, 3}, {1, 2, 3}, K} and $\mathcal{I}$ = {∅, {2}}. It is clear that {1, 2, 3, 4} is α-m$\mathcal{I}$-open but not micro-open.*

Remark 4.7. *It is clear that m$\mathcal{I}$-open and micro open are independent concepts as it is shown by the following example.*

Example 4.8. *Let K, $\mathcal{N}$, μ, $\mathcal{M}$ be defined as an Example 3.4 (1). Then m$\mathcal{I}$-open sets are ϕ, {2}, {4}, {2, 4}, K ; micro open sets are ϕ, {1}, {2, 4}, {1, 2, 4}, K. It is clear that {1} is micro open sets but not m$\mathcal{I}$-open set. Also it is clear that {2} is m$\mathcal{I}$-open sets but not micro open sets.*

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Remark 4.9. *From the above discussions and known results in [[1], [5], [6]] i obtain the following diagram where $A \to B$ represents A implies B, but not conversely.*

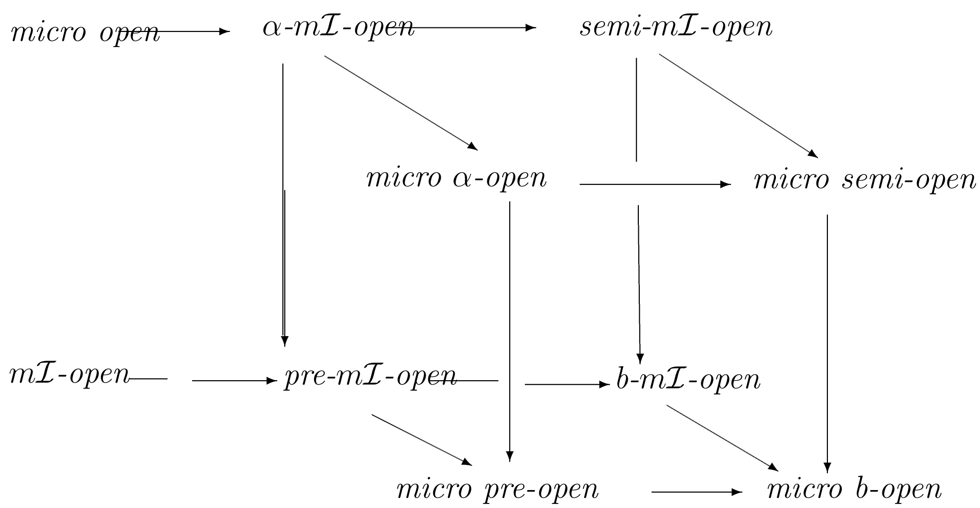

Diagram-III

b-m$\mathcal{I}$-open $\to$ *micro b-open*

$\downarrow$ $\downarrow$

β-m$\mathcal{I}$-open $\to$ *micro β-open*

Diagram-IV

None of the above implications is reversible as shown in the remaining examples and in the related paper [[1], [5], [6]].

Proposition 4.10. *Let $(K, \mathcal{N}, \mathcal{M}, \mathcal{I})$ be an micro ideal topological space and $A \subseteq K$. Then the following hold: (a) If $\mathcal{I} = \{\phi\}$, then*

(1) *m$\mathcal{I}$-open, pre-m$\mathcal{I}$-open and micro pre-open are all equivalent.*

(2) *A is semi-m$\mathcal{I}$-open if and only if A is micro semi-open.*

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(3) *A is β-m$\mathcal{I}$-open if and only if A is micro β-open.*

(b) If $\mathcal{I} = P(K)$, then A is β-m$\mathcal{I}$-open if and only if A is micro semi-open.

(c) If $\mathcal{I} = N$, then A is β-m$\mathcal{I}$-open if and only if A is micro β-open, where N is the ideal of all nowhere dense sets.

Proof. (a) If $\mathcal{I} = \{\phi\}$, then A^*_m = m-cl(A^*_m) = m-cl(A) for any subset A of K and hence m-cl*(A) = A ∪ A^*_m = A ∪ m-cl(A) = mcl(A) ∪ m-cl(A) = m-cl(A). Therefore, we obtain A^*_m = m-cl(A) = m-cl*(A). Thus, (1), (2) and (3) follow immediately.
(b) Let $\mathcal{I}$ = P(K) then $A^*_m = \phi$ (See Example 3.2 (2)) for any subset A of K. Therefore, we have m-cl*(m-int(m-cl*(A))) ⊆ m-cl*(m-int(A ∪ A^*_m)) ⊆ m-cl*(m-int(A ∪ m-cl(A))) ⊆ m-cl*(m-int(m-cl(A) ∪ m-cl(A)))⊆ m-cl*(m-int(m-cl(A))) ⊆ ((m-int(m-cl(A)))$^*_m$ ∪ (m-int(m-cl(A)) ⊆ m-cl(m-int(m-cl(A))) = m-cl(m-int(A)) = Mic-cl(Mic-int(A)). Thus, β-m$\mathcal{I}$-openness and micro semi-openness are equivalent.
(c) By Proposition 4.2 (3), every β-m$\mathcal{I}$-open set is micro β-open. If $\mathcal{I}$ = N, then it is well-known that A^*_m = Mic-cl(Mic-int(Mic-cl(A))). Therefore, if A is micro β-open we obtain A ⊆ Mic-cl(Mic-int(Mic-cl(A))) = A^*_m = m-cl*(A) and hence A ⊆ Mic-cl(Mic-int(Mic-cl(A))) = m-cl(m-int(m-cl(A))) = ((m-int(m-cl(A)))$^*_m$ ∪ (m-int(m-cl(A)) = m-cl*(m-int(m-cl(A))) = m-cl*(m-int(m-cl(A) ∪ m-cl(A))) = m-cl*(m-int(A ∪ m-cl(A))) = m-cl*(m-int(A ∪ A^*_m)) = m-cl*(m-int(m-cl*(A))). □

Definition 4.11. *Let A subset A of an micro ideal topological space (K, $\mathcal{N}$, $\mathcal{M}$, $\mathcal{I}$) is said to be micro regular closed (briefly, regular-m$\mathcal{I}$-closed) if A = (m-int(A))$^*_m$.*

Theorem 4.12. *For a subset A of an micro ideal topological space (K, $\mathcal{N}$, $\mathcal{M}$, $\mathcal{I}$), the following properties hold.*

(1) *Every regular-m$\mathcal{I}$-closed set is α^*-m$\mathcal{I}$-open and semi-m$\mathcal{I}$-open.*
(2) *Every regular-m$\mathcal{I}$-closed set is m$\star$-perfect.*

Proof. (1) Let A be regular-$m\mathcal{I}$-closed set. Then, we have m-cl*(m-int(A)) = m-int(A) $\cup$ (m-int(A))$^*_m$ = m-int(A) $\cup$ A = A. Thus, m-int(m-cl*(m-int(A))) = m-int(A) and A $\subset$ m-cl*(m-int(A)). Therefore, A is α^*-$m\mathcal{I}$-open and semi-$m\mathcal{I}$-open.
(2) Let A be regular-$m\mathcal{I}$-closed set. Then we have A = (m-int(A))$^*_m$. Since m-int(A) $\subset$ A, (m-int(A))$^*_m$ $\subset$ (A)$^*_m$ by Theorem 3.3 (1), (3), (4), (5). Then we have A = (m-int(A))$^*_m$ $\subset$ (A)$^*_m$. On the other hand by Theorem 3.3 it follows from A = (m-int(A))$^*_m$ that (A)$^*_m$ = ((m-int(A))$^*_m$)$^*_m$ $\subset$ (m-int(A))$^*_m$ = A. Therefore, we obtain A = (A)$^*_m$. Hence, A is m$\star$-perfect. □

Example 4.13. *Let K = {1, 2, 3, 4} with K/ R= {{2}, {4}, {1, 3}} and X = {1, 3}. The nano topology $\mathcal{N}$ = {ϕ, {1, 3}, K}. If μ = {4} then the micro topology $\mathcal{M}$ = {ϕ, {4}, {1, 3}, {1, 3, 4}, K} and the ideal $\mathcal{I}$ = {$\emptyset$, {3}, {4}, {3, 4}}. Then (i) A = {1, 2}. Then A is an α^*-$m\mathcal{I}$-open set which is not regular-$m\mathcal{I}$-closed. For A = {1, 2} $\subset$ K, since m-int(A) =ϕ, (m-int(A))$^*_m$ = ϕ and hence m-cl*(m-int(A)) = m-int(A) $\cup$ (m-int(A))$^*_m$ =ϕ. Thus, we have m-int(m-cl*(m-int(A))) = ϕ = m-int(A) and hence A is an α^*-$m\mathcal{I}$-open set. On the other hand, since (m-int(A))$^*_m$ = $\phi \neq$ {1, 2} = A, A is not regular-$m\mathcal{I}$-closed.*
(2) A = {1, 3}. Then A is semi-$m\mathcal{I}$-open set which is not regular-$m\mathcal{I}$-closed. For A = {1, 3} $\subset$ K, since m-int(A) = {1, 3}, (m-int(A))$^*_m$ = {1, 2, 3} and hence m-cl*(m-int(A)) = m-int(A) $\cup$ (m-int(A))$^*_m$ = {1, 2, 3} $\supset$ {1, 3} = A. Hence, A is a semi-$m\mathcal{I}$-open set. On the other hand, (m-int(A))$^*_m$ = {1, 2, 3} $\neq$ {1, 3} = A and hence A is not regular-$m\mathcal{I}$-closed.

Example 4.14. *Let K = {1, 2, 3} with K/ R= {{1}, {2, 3}} and X = {1}. The nano topology $\mathcal{N}$ = {ϕ, {1}, K}. If μ = {1, 2} then the micro topology $\mathcal{M}$ = {ϕ, {1}, {1, 2}, K} and the ideal $\mathcal{I}$ = {$\emptyset$, {1}, {2}, {1, 2}}. A= {3}. Then A is m$\star$-perfect but not regular-$m\mathcal{I}$-closed. For A = {3} $\subset$ K, (A)$^*_m$ = {3} = A and hence A is*

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$m\star$-perfect. On the other hand, since m-int(A) $= \phi$ and $\phi \in \mathcal{I}$ we have (m-int(A))$^*_m$ $=(\phi)^*_m = \phi \neq \{3\} = A$. Hence A is not regular-$m\mathcal{I}$-closed.

Corollary 4.15. *Every regular-$m\mathcal{I}$-closed set is $m\star$-closed and $m\star$-dense in itself.*

Proof. The Proof is obvious from Theorem 4.12. □

Theorem 4.16. *In an micro ideal topological space (K, $\mathcal{N}$, $\mathcal{M}$, $\mathcal{I}$), every regular-$m\mathcal{I}$-closed set is micro regular closed.*

Proof. Let A be any regular-$m\mathcal{I}$-closed set. Then we have A = (m-int(A))$^*_m$ = m-cl((m-int(A))$^*_m$) = m-cl(m-int(A)) = Mic-cl(Mic-int(A)). Hence, A is micro regular closed set. □

Example 4.17. *Let K, $\mathcal{N}$, μ, $\mathcal{M}$ be defined as an Example 4.13. A= $\{2, 4\}$. Then A is micro regular closed set which is not regular-$m\mathcal{I}$-closed. For A $= \{2, 4\} \subset K$, since Mic-int(A) $= \{4\}$, Mic-cl(Mic-int(A)) = Mic-cl($\{4\}$) $= \{2, 4\} = A$ and A is micro regular closed set. On the other hand, since m-int(A) $= \{4\}$ and $\{4\} \in \mathcal{I}$, we have (m-int(A))$^*_m$ $= (\{4\})^*_m = \phi \neq \{2, 4\} = A$ and hence A is regular-$m\mathcal{I}$-closed.*

Remark 4.18. *From the above discussions i obtain the following diagram where A $\rightarrow$ B represents A implies B, but not conversely.*

$m\star$ closed		α^*-$m\mathcal{I}$-open		α-$m\mathcal{I}$-open
$\uparrow$		$\uparrow$		$\downarrow$
$m\star$-perfect	$\longleftarrow$	regular-$m\mathcal{I}$-closed	$\longrightarrow$	semi-$m\mathcal{I}$-open
$\downarrow$		$\downarrow$		$\downarrow$
$m\star$ dense in itself		micro regular closed	$\longrightarrow$	micro semi-open

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Diagram-V

Conclusion

Every year different type of topological spaces are introduced by many topologist. Now a days available topologies are ideal topology, bitopology, fuzzy topology, fine topology, nano topology, nano ideal topology and so on. micro topology introduced and studied by Chandrasekar [1]. In this paper we introduced a new concept of spaces called micro ideal topological spaces and investigate the relation between micro topological space and micro ideal topological spaces. We define some closed sets in these spaces to establish their relationships. Basic properties and characterizations related to these sets are studied. We introduced and studied the new concepts called micro regular open set, micro π-open set in micro topological spces and also the new concepts called $m\mathcal{I}$-open, α-$m\mathcal{I}$-open, pre-$m\mathcal{I}$-open, semi-$m\mathcal{I}$-open, b-$m\mathcal{I}$-open, β-$m\mathcal{I}$-open, regular $m\mathcal{I}$-closed, which are simple forms of micro open sets in an micro ideal topological spaces. Also we characterize the relations between them and the related properties. In future, we have extended this work in various micro ideal topological fields with some applications.

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